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Comparing the Logarithmic Transformation and the Box-Cox Transformation for Individual Tree Basal Area Increment Models

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Individual tree growth models are increasingly being used in silviculture scenario simulation at the stand level or in forecasts of wood supply on a large scale, and there is a correspondingly substantial number of published diameter increment models. In most cases, the relationship between individual tree basal area increment or diameter increment and covariables was described by a linear regression. In doing so, the logarithmic transformation for left-sided variable transformation was used exclusively to meet the assumptions of regression analysis. The Box-Cox transformation is one alternative that has scarcely been used to date in forest growth modeling. The two transformation approaches were compared using a simple individual tree basal area increment model with four tree species. The results were as follows: (1) the Box-Cox transformation yielded a better residual structure of the models by reducing the skew; (2) the transformation bias is smaller using the Box-Cox transformation; (3) the mean squared error of estimation is smaller with the Box-Cox transformation; and (4) the Box-Cox transformation leads to systematically higher estimated values than logarithmic transformation. On the basis of the results presented here, it is recommended that the Box-Cox transformation should be considered as a viable alternative in statistical modeling in forestry and in other fields as well if the transformation of variables is required.

Keywords: individual tree growth, generalized additive model, forest inventory data

The accurate prediction of stand development over time is essential for forest planners. In Europe, forest management traditionally relied on yield tables for making such predictions. However, for the management of structurally diverse forests, yield tables are increasingly unsuitable as a planning instrument (Monserud and Sterba 1996). For this reason, individual tree growth models were developed to support forestry planning and management (e.g., Stage 1973, Wykoff et al. 1982, Pretzsch 1992, Hasenauer 1994, Nagel 1999). The advantage of these models in comparison to stand-based models lies in their greater flexibility. Thus, prognoses are possible for heterogeneous forest stands in which the composition of tree species, age, and vertical structure, as well as spatial occupation patterns can vary considerably. One important element in individual tree growth modeling is the growth prediction of the tree dbh using linear or nonlinear regression techniques. In both approaches, the dependent variable is either the actual diameter increment or the basal area increment of the individual tree, which is then converted to a diameter increment. The predictor variables fall into three categories: the characteristics of individual trees such as dbh, basal area, height, crown parameters, and age; competition at the stand or tree level; and site variables such as water and nutrient supply, altitude, direction of slope, temperature regime, and others.

Several authors use nonlinear regression to estimate increment as a function of individual tree attributes (e.g., Colbert et al. 2004) or as a function of tree attributes and competition indices (e.g., Yang et al. 2009). Expanded models contain tree attributes, competition indices, and several site parameters as predictors (Pretzsch et al. 2002, Weiskittel et al. 2007, Bollandsas and Naesset 2009). More often, however, increment is estimated by means of linear regression. In Cole and Lorimer (1994), Nagel (1999), Jögiste (2000), Mailly et al. (2003), and Ledermann (2010), individual tree attributes and competition indices are used as predictors. Expanded models containing additional site parameters are presented in numerous studies (e.g., Wykoff 1990, Monserud and Sterba 1996, Uzoh and Oliver 2008, Lhotka and Loewenstein 2011, Pokharel and Dech 2012).

In contrast to nonlinear approaches, the application of linear regression often requires a transformation of the dependent variables to meet model assumptions. That means homoscedasticity and

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This article uses metric units; the applicable conversion factors are: centimeters (cm): 1 cm = 0.39 in.; meters (m): 1 m = 3.3 ft; square meters (m^2): 1 m^2 = 10.8 ft^2; kilometers (km): 1 km = 0.6 mi; hectares (ha): 1 ha = 2.47 ac.

Table 1. Summary statistics of the modeling data.

Species	N_p	N_t	Dbh		Age		BAL	
			Mean	Range	Mean	Range	Mean	Range
			(cm)		(yr)		(m^2/ha)	
Oak	693	1,646	35.9	10.0–153.2	88	17–269	13.1	0.0–77.1
Beech	927	3,137	32.5	10.0–149.0	85	12–279	18.7	0.0–99.1
Spruce	997	3,162	32.2	10.0–96.8	53	9–176	17.4	0.0–74.8
Pine	1,265	4,351	31.0	10.0–80.0	54	11–196	13.0	0.0–64.0

N_p, number of subplots; N_t, number of measured trees.

a normal distribution of the residuals. In many cases, however, the most important reason for the preference for the linear approach is that the relationships between the transformed response variables and the predictors can be approximated by linear model effects. However, transformation is at the expense of dealing with a bias after retransformation to the original scale of measurement (e.g., Smith 1993). In modeling diameter or basal area increment by linear regression, the response variable is usually logarithmically transformed (see the literature citations above).

In this article, the Box-Cox transformation (Box and Cox 1964) is evaluated as an alternative method. To date, this type of transformation has rarely been applied in forest growth modeling. The Box-Cox transformation belongs to the family of exponential transformations and was analyzed in detail by Sakia (1990, 1992), whereas in Garcia (1983), Eastaugh and Hasenauer (2011), Mønness (2011), and Serinaldi et al. (2012), forestry-related examples of its use are given. In this approach, in contrast to logarithmic transformation, a transformation parameter that is dependent on the distribution of the data is at first estimated. This makes the Box-Cox transformation more flexible than other methods because the conversion into the transformed scale is specifically a function of the distribution of the data. In this study, we examine whether this flexibility also affects the goodness of fit and the predictive quality of statistical models. For this purpose, the Box-Cox-transformation and the logarithmic transformation are compared using an individual tree basal area increment model. The model fitting for both transformation approaches is done using repeated individual tree measurement data for four tree species. The data were acquired over the course of a large-scale forest inventory in Northwest Germany. The following three issues will be addressed: a comparison of the goodness-of-fit statistics and examination of transformed and retransformed model residuals; a comparison of the mean squared error (MSE) by cross-validation; and an analysis of the predicted diameter growth.

Materials and Methods

Data

The data used for model fitting were drawn from repeated individual tree measurements from the first and second National Forest Inventory (NFI) in the federal German states of Lower Saxony and Schleswig-Holstein. The first inventory (NFI 1) was carried out between 1986 and 1989, and this was repeated between 2001 and 2003 (NFI 2). The NFI is a cluster sample with permanent sample plots. The sample plots in the east and south of Lower Saxony are arranged in a 4-km square grid, which is based on the Gauss-Krüger coordinate system. In the lowlands of western Lower Saxony the grid is 2.83 × 2.83 km in size, and in Schleswig-Holstein it is 2 × 2 km. Each sample plot consists of a square with sides of length 150 m, where the southwest corner of each square is the intersection point of the grid. If one corner of the square is forested, that corner becomes the center of a subplot and data for different objects are surveyed. Trees (dbh ≥10 cm in NFI 1; dbh ≥7 cm in NFI 2) are surveyed by means of the angle count sampling method (Bitterlich sampling) with a basal area factor of 4 m^2/ha. The tree species, dbh, age, and the base-of-the-trunk coordinates are recorded for each monitored tree (Bundesministerium für Ernährung und Landwirtschaft 2014). This study is based on data from 3,045 NFI subplots with repeated individual tree measurements. Four tree species were studied: oak (*Quercus robur* L.), beech (*Fagus sylvatica* L.), spruce (*Picea abies* L.), and pine (*Pinus sylvestris* L.) (Table 1).

Model Construction

In this study, the diameter increment of individual trees was indirectly modeled through the relationship of basal area increment to dbh, age, and competitive status. The basal area in larger trees (BAL) (e.g., Wykoff 1990) was used as a competition index. To take into account any occurrence of nonlinearity in the effects of the covariables, a generalized additive model (GAM) was used. This method is implemented in the R library *mgcv* (Wood 2006, R Core Team 2015). The 5-year basal area increment *ig5* (cm^2) of a tree is expressed through the equation

$$ig5 + c = \alpha + f_1(\text{dbh}) + f_2(\text{age}) + f_3(\text{bal}) \tag{1}$$

where c is a constant to eliminate negative values, α is the intercept, and f_n are smoothing functions defined as "thin plate" regression splines. In line with other studies using similar types of data (Monserud and Sterba 1996, Andreassen and Tomter 2003, Pukkala et al. 2009), the model was formulated without random effects. In view of the high number of systematically distributed sample plots, the comparatively low number of trees monitored per plot (on average 4.5 trees per plot over all sampled tree species) and the lack of repeated measurements of individual trees (only one recorded increment per tree), the probability of a grouped structure in the data was deemed to be negligible.

Before model fitting, the dependent variable from Equation 1 was transformed using two methods. For the logarithmic transformation (natural logarithm), the relationship between the original and the transformed scales of a variable is given by

$$y' = \ln(y) \tag{2}$$
$$y = \exp^{y'}$$

The second method used is the Box-Cox-transformation (Box and Cox 1964). The relationship between the transformed and the original data is

$$\left.\begin{aligned} y' &= \frac{y^{\lambda} - 1}{\lambda} \\ y &= (y' \cdot \lambda + 1)^{\frac{1}{\lambda}} \end{aligned}\right\} \quad \lambda \neq 0 \tag{3}$$

$$\left.\begin{array}{l} y' = \ln(y) \\ y = \exp^{y'} \end{array}\right\} \quad \lambda = 0$$

The Box-Cox transformation is equal to the logarithmic transformation when $\lambda = 0$. The value of the transformation parameter λ can be estimated iteratively using the maximum-likelihood method, where the log-likelihood term is defined as

$$L(\lambda) = k - \frac{n}{2}\mathrm{SSE}(z^{(\lambda)}) \tag{4}$$

with k as an integration constant, $z^{(\lambda)} = y^{(\lambda)}/y_m^{(\lambda-1)}$, and with y_m as the geometric mean of the response. SSE is the sum of the squared residuals of the regression $z^{(\lambda)}$ (Venables and Ripley 2002, p. 171). Sakia (1992) gives further information about the mathematical background. The value of λ is optimal when L is at its maximum.

One consequence of the use of transformations is that transforming the predicted values back again (retransforming) leads to systematic distortions of estimates in the original scale (bias). There are diverse adjustment factors available to help avoid this effect, especially for the logarithmic transformation (e.g., Sprugel 1983, Smith 1993). To facilitate comparison, a single adjustment factor suggested by Snowdon (1991) was used here for both transformation methods, which is calculated as the quotient of the sum of the observed values and the predicted values. This approach has been used in various studies (e.g., Nyström and Kexi 1997, Condes and Sterba 2005, 2008, Adame et al. 2008).

$$\gamma = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} \hat{y}_i} \tag{5}$$

with n being the number of observations, y_i being the observed values, and $\hat{y}_i$ being the estimated values in retransformed scale. Multiplying the retransformed values by γ ensures that the mean predicted basal area increment corresponds to the mean observed basal area increment. γ simultaneously serves as a measure of the transformation bias. The mean of the estimated values $\hat{y}_i$ shows no bias when $\gamma = 1$.

Model Evaluation

The coefficient of determination was used as the criterion for goodness of fit

$$R^2 = 1 - \left(\frac{\sum_{i=1}^{n}(y_i' - \hat{y}_i')^2}{\sum_{i=1}^{n}(y_i' - \bar{y}')^2}\right) \tag{6}$$

with y_i' and $\hat{y}_i'$ as the observed and estimated values in the transformed scale.

In a second step, the residual structure, in both transformed and retransformed scale, was graphically analyzed. The skew of the distribution of the transformed residuals was calculated. This quantifies the type and extent of any asymmetry in the distribution and is independent of the unit of measurement.

$$\mathrm{skew} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i' - \hat{y}_i'}{S}\right)^3 \tag{7}$$

with s as the SD of the transformed residuals. If the data are exactly normally distributed, the skew will be zero. If there is a skew to the right, the value will be negative, whereas a positive value indicates a skew to the left.

The two transformation methods were compared using a cross-validation of the MSE in the retransformed scale. The MSE is a measure of the estimation accuracy of the model and is given by

$$MSE = \sum_{i=1}^{n}\frac{(y_i - \hat{y}_i \cdot \gamma)^2}{n} \tag{8}$$

For cross-validation, 75% of the observations for each tree species were randomly selected from the complete available data set. A parameterization of the basal area increment model was carried out on this subset of the data using both transformation methods. This process was repeated 100 times, enabling a comparison of the variability of the MSE by the distributions produced.

Results

For both transformation methods, the effect of each covariable on the basal area increment is significant, as indicated by the P values (Table 2). The fitted model for oak using the Box-Cox transformation shows a slightly higher R^2 value, whereas for beech the difference between the two transformation methods is marked. For spruce and pine, there is no difference in R^2 between the two types of transformations. The adjustment factor γ quantifies the relative transformation bias. The values for γ obtained after a logarithmic transformation are, depending on tree species, between 1.123 (pine) and 1.147 (beech), meaning that the observed values are on average 12 to 15% greater than the retransformed estimates. Use of the Box-Cox transformation yields a considerably smaller transformation bias, i.e., 8% for oak and beech ($\gamma = 1.081$ and $\gamma = 1.084$) and 5% for pine and spruce ($\gamma = 1.052$ and $\gamma = 1.053$). The estimates of the confidence interval for the transformation parameter λ for all tree species do not allow for the value zero, meaning, according to the likelihood estimate for λ in Equation 4, that the logarithmic transformation is not appropriate.

The effects of the covariables are nonlinear and show similar patterns for all tree species and for both types of transformation. As an example, the results for Norway spruce are displayed (Figure 1). It is apparent that the dbh has a positive effect on increment. Age and competition, on the other hand, always have a negative effect. Because of the extensive data set, the confidence intervals across the whole range of values indicate robust estimates, with increasing scattering evident only in the peripheries.

The analysis of the model residuals for normal distribution was carried out using a graphical comparison of empirical and theoretical quantiles (Q-Q plot). When the logarithmic transformation was used, the Q-Q plots of the residuals for all tree species are curved, with clear divergence in the lower quantiles (Figure 2). This pattern, as well as the negative skew values, points to a distribution skewed to the right, which means that the contribution of the negative values

Table 2. Model output of GAM fits.

Transformation	Species	Term	*edf*	*P* value	R^2	γ	λ (CI: 5%; 95%)
Logarithmic	Oak	*f*1 (dbh)	8.597	0.000	0.598	1.145	
		*f*2 (age)	8.331	0.000			
		*f*3 (bal)	3.149	0.000			
	Beech	*f*1 (dbh)	8.326	0.000	0.535	1.147	
		*f*2 (age)	8.559	0.000			
		*f*3 (bal)	4.372	0.000			
	Spruce	*f*1 (dbh)	7.751	0.000	0.303	1.146	
		*f*2 (age)	6.569	0.000			
		*f*3 (bal)	3.100	0.000			
	Pine	*f*1 (dbh)	8.082	0.000	0.403	1.123	
		*f*2 (age)	8.631	0.000			
		*f*3 (bal)	5.071	0.000			
Box-Cox	Oak	*f*1 (dbh)	8.064	0.000	0.604	1.084	0.372 (0.353;0.382)
		*f*2 (age)	8.380	0.000			
		*f*3 (bal)	3.166	0.000			
	Beech	*f*1 (dbh)	6.446	0.000	0.644	1.081	0.388 (0.380;0.396)
		*f*2 (age)	8.535	0.000			
		*f*3 (bal)	4.261	0.000			
	Spruce	*f*1 (dbh)	7.619	0.000	0.303	1.052	0.563 (0.550;0.576)
		*f*2 (age)	4.399	0.000			
		*f*3 (bal)	2.611	0.000			
	Pine	*f*1 (dbh)	6.686	0.000	0.405	1.053	0.514 (0.495;0.533)
		*f*2 (age)	8.647	0.000			
		*f*3 (bal)	1.000	0.000			

edf, estimated degrees of freedom for the model terms (degree of smoothing); γ, parameter for bias correction (Equation 5); λ, transformation parameter (Equation 3); CI, confidence interval.

in the lower quantiles is greater than expected in a normal distribution. Use of the Box-Cox transformation leads to a much better residual structure, with the reduction of skew lying between 65% (beech) and 78% (spruce). A divergence from a straight line in the lower quantile area is still present. However, it is less pronounced for all tree species. The plots are more consistent, with marginal divergence from the theoretical quantiles.

Validation of the predictive quality of both methods is carried out in the original scale of measurement. The mean of the retransformed residuals after bias correction was modeled as a function of the predicted values using a smoothing function (loess-smoothing) (Figure 3). Both transformation methods estimate the 5-year basal area increment on average almost without bias, with merely a slight tendency to overestimate the higher predicted values for the logarithmic transformation. The residuals show a diagonal pattern in the negative value range, which means that the models only estimate positive increment.

The results of the cross-validation show significant differences between the two transformation methods for the MSE of the models (Figure 4). The median of the response variables transformed by the logarithmic method are 5% higher for oak and 3.5% higher for beech. For spruce, the difference is 1.2%. It becomes clear that use of the Box-Cox transformation leads to improved estimate accuracy.

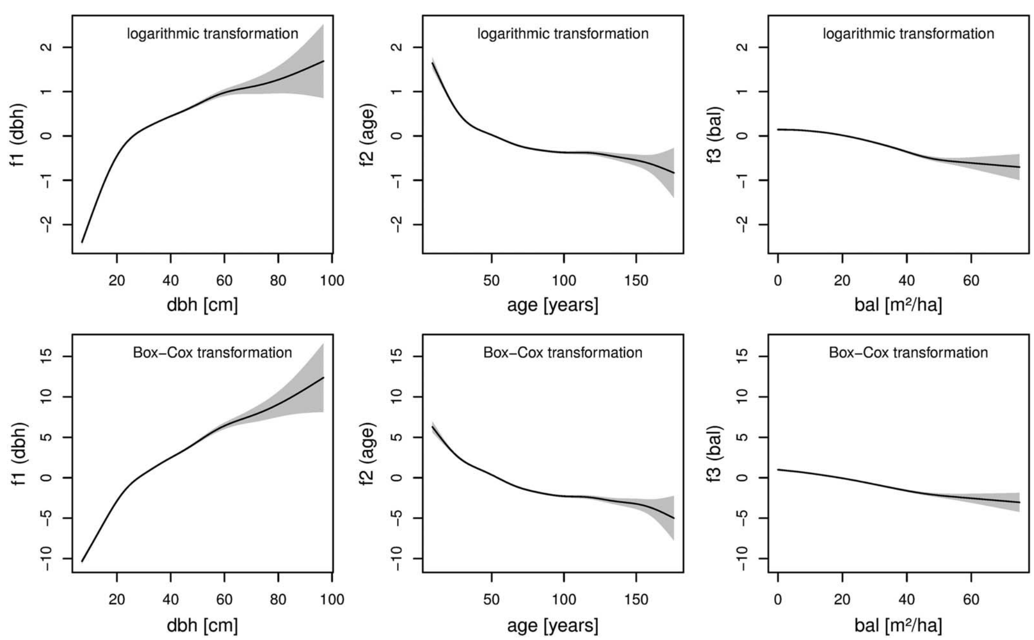

Figure 1. Effects of the predictors on basal area increment for spruce. Shaded areas represent 2 times the SE of the expectation value.

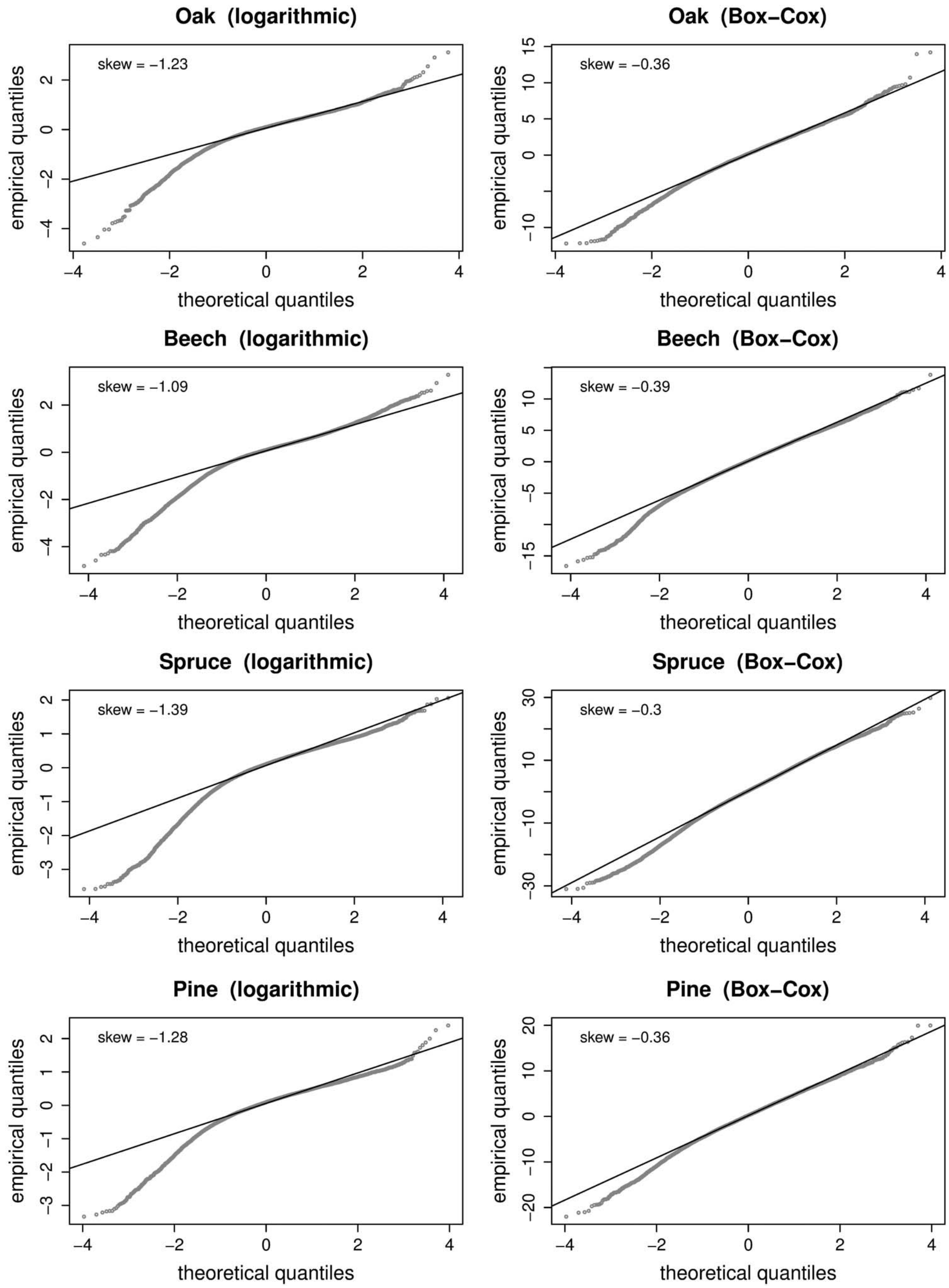

Figure 2. Q-Q plots of the model residuals after logarithmic transformation (left column) and after Box-Cox transformation of the dependent variable (right).

The effect of the two transformation methods in model application was further evaluated. For this purpose, diameter increment of a fictitious tree, which was growing under average competition (Table 1), was predicted for a period of 30 years (Figure 5). The difference between the curves increases with elapsed time, the values of the log model always being lower than those for the Box-Cox model. At the end of the simulation time period, the diameter difference lies between 3 and 4 cm, depending on tree species.

Discussion

Choice of Model

The purpose of the present study is to carry out a comparison of two transformation approaches. By using easily determined covariables, an attempt is made to reach a compromise between model plausibility, robustness, and flexibility. With the basal area increment model, one cannot always assume a strict linear effect, even for the transformed explanatory variable (e.g., Schröder et al. 2002, Pukkala et al. 2009, Pokharel and Dech 2012). For this reason, a generalized additive model was chosen for this study. The link function does not link the response variable with the predictor over the entire value range. Instead, it is made up of many locally adapted smoothing terms, so that genuine nonlinearity in the covariable effects is taken into account (Figure 1).

The dbh has been identified as a significant term in many previous studies (Wykoff 1990, Hökkä et al. 1997, Mailly et al. 2003) and was therefore also incorporated in the model in this study. Site

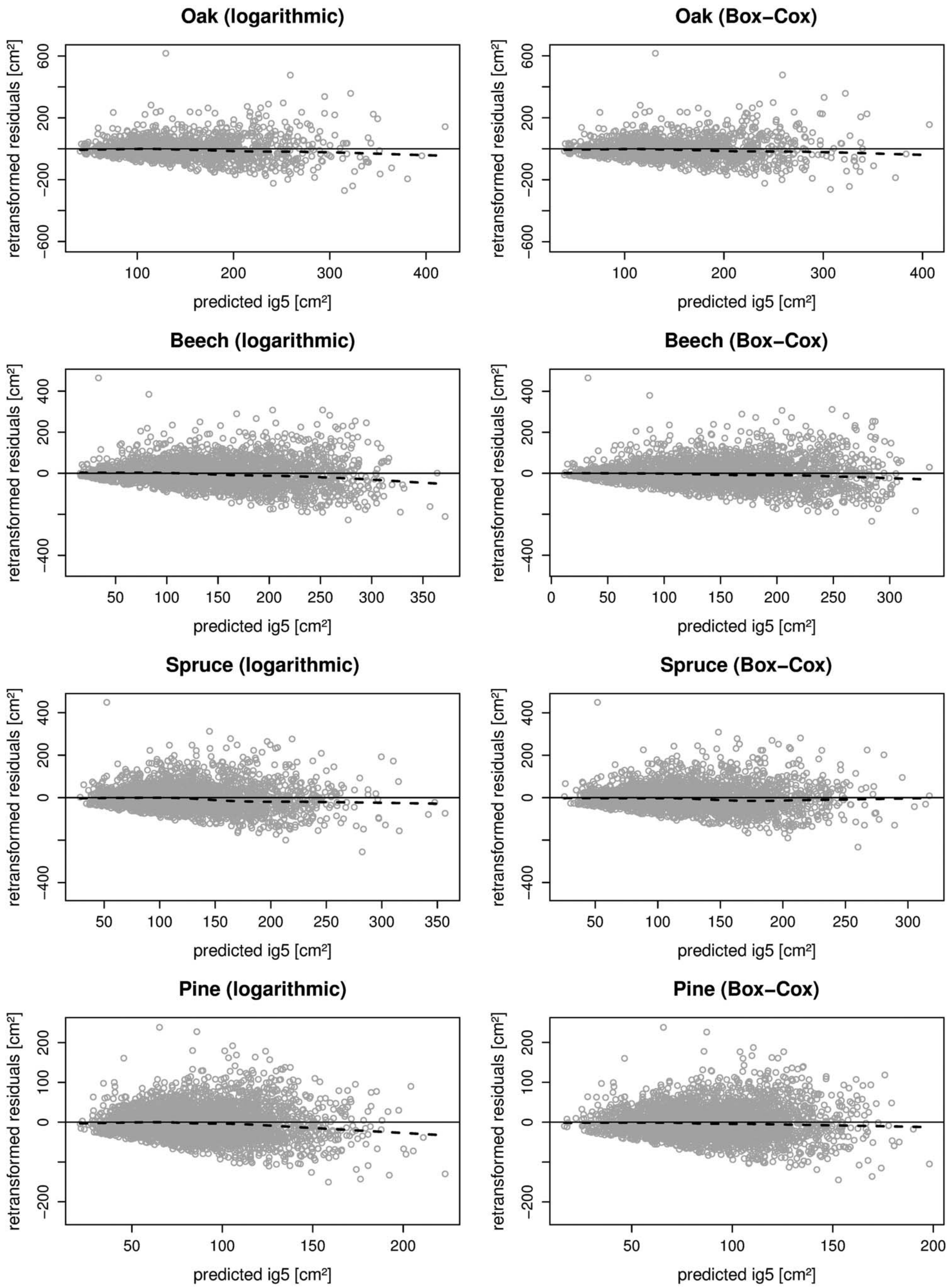

Figure 3. Residuals plotted against predicted values after retransformation. The dashed lines represent the mean residuals as a function of the predicted values.

characteristics are further deciding factors in explaining the site-determined difference in individual tree growth. Including these factors would increase the ecological plausibility and perhaps also the accuracy of the prediction. However, a higher volume of often difficult to determine input information is required. This seemed impractical for the purpose of this study. The site index is often used as an alternative to directly measured site variables (e.g., Mailly et al. 2003, Uzoh and Oliver 2008). However, because relatively few individual tree height measurements were taken in the sample plots, a reliable calculation of the site index using the available data was not possible. To build a site effect into the model, tree age was used as a further covariable. Site influence could then be indirectly inferred by the dbh/age ratio. Assuming a constant competitive status over time, a thicker tree is evidence of site conditions that developed more favorably for the growing tree than those that prevailed for a thinner tree of the same age.

To describe the effect of competition on individual trees, the BAL was incorporated in the model. In contrast to a diameter percentile, the BAL exhibits a realistic relationship to changes in the competitive regime. For instance, the diameter percentile for a reference tree decreases if another tree with a smaller diameter is removed (Wykoff 1990, Monserud and Sterba 1996). This inconsistency does not arise when the BAL is used because the value would remain unchanged in this case.

All effects of parameterized models could be robustly estimated, indicated by the P values (Table 2) and confidence intervals of smoothing splines (Figure 1). The adjusted coefficients of determination of the models were, despite the comparatively small number

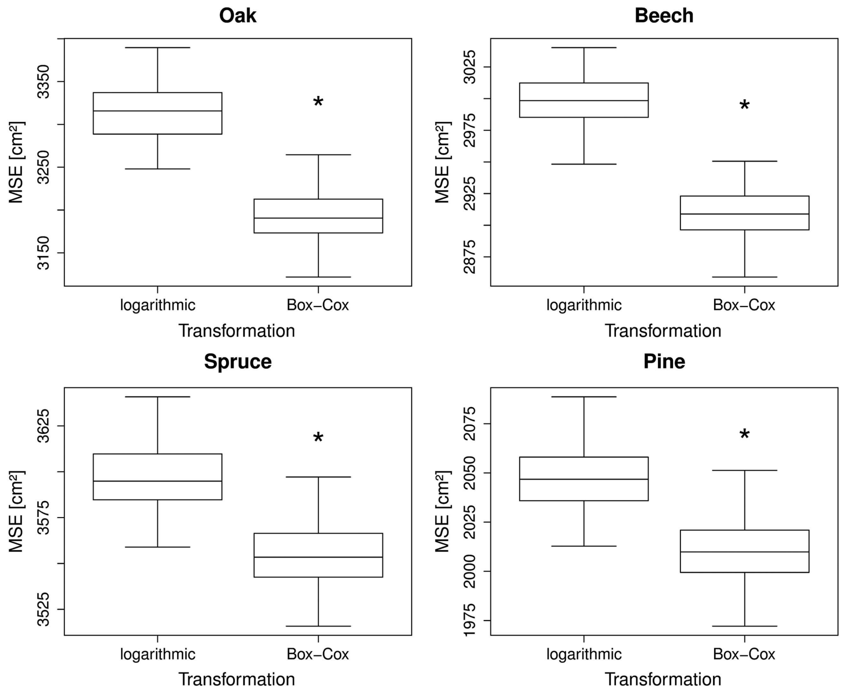

Figure 4. Results of cross-validation (n = 100) of the MSE. *Significant lower values of distribution mean values (paired *t*-test, $\alpha = 0.05$, 99 *df*). The horizontal line within the box shows the median; the bottom and the top of the box show the 25th and 75th percentiles. The vertical lines show the maximum and minimum values.

of covariables used, similar to those from the studies of Monserud and Sterba (1996), Sterba et al. (2002), Andreassen and Tomter (2003), or Pukkala et al. (2009).

Transformation Methods

One general aspect that should be addressed by the modeler is whether a variable transformation is necessary. For growth modeling in particular, there are good reasons to use nonlinear functions that describe the change in size of an individual or population over time (Zeide 1993). In contrast to linear models or GAM, these theoretical models have an underlying hypothesis associated with cause or function of the phenomenon described by the response variable. Therefore, model parameters are meaningful and more easily interpretable, and predictions that involve

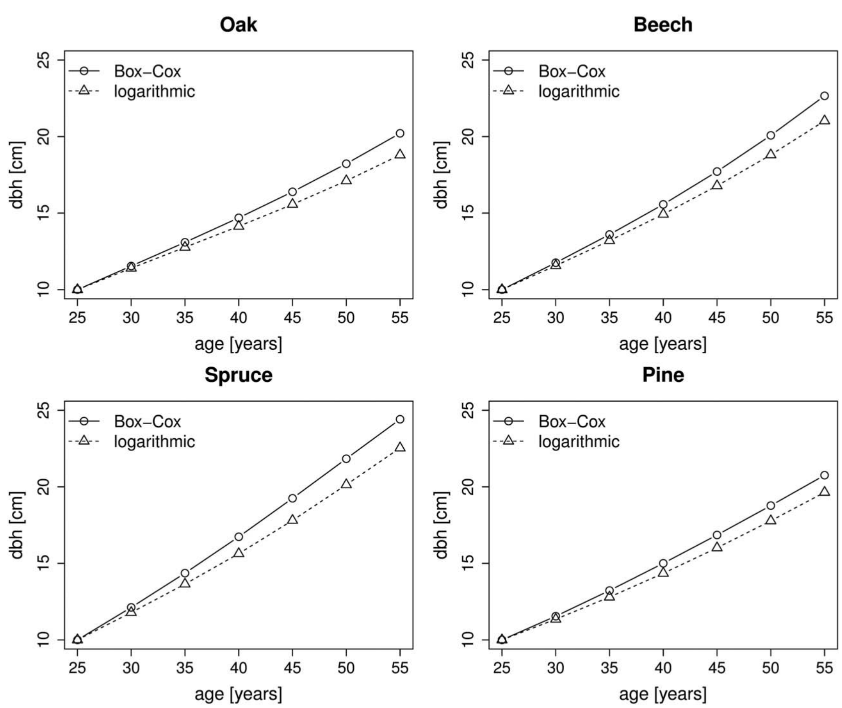

Figure 5. Predicted diameter corresponding to the models presented in Table 1. Starting values for the fictive tree are dbh_0 = 10 cm, age_0 = 25 years, and bal_0 = mean values for each species according to Table 1 (set as constant).

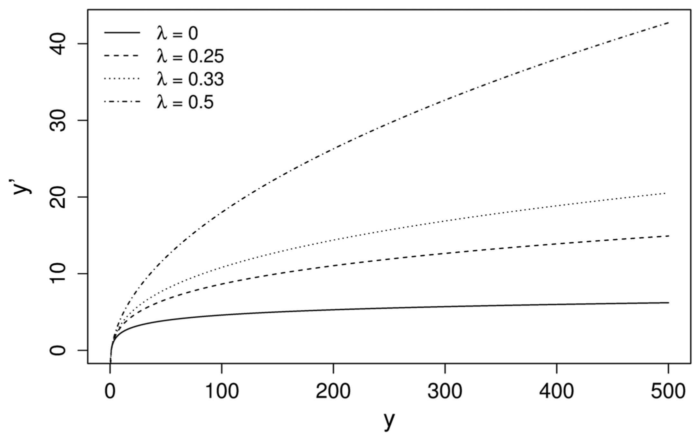

Figure 6. Effect of parameter λ in Box-Cox transformation on the rescaling of a variable *y* (Equation 3).

extrapolations beyond the range of data are more reliable (Fekedulegn et al. 1999). Features of biological processes, such as asymptotic behavior, can often be expressed more accurately using nonlinear models. Another advantage of nonlinear curve fitting is its economical use of parameters. For describing a growth process, a competing polynomial model would probably need more parameters than a nonlinear growth function. However, this problem can also be avoided by using GAM instead of linear regression. One disadvantage in comparison to linear regression or GAM is that there is no analytical solution for parameter estimation and that convergence of iterative optimization procedures can depend critically on having good stating values. If multiple independent variables are to be used for prediction, modifications of the given nonlinear function are required. Because growth functions usually only allow for one predictor (Burkhart and Tomé 2012, p. 114–115), for example, age, model parameters then must be expressed as functions of the remaining predictors (e.g., Temesgen and von Gadow 2003). Another possibility to deal with multiple predictors is to extend the base model by developing a multiplicative modifier function that contains the additional predictors (e.g., Pretzsch et al. 2002, Bollandsas and Naesset 2009). For relatively extensive models with a large number of predictors (e.g., Monserud and Sterba 1996), linear models or GAM may be more suitable because of the simpler specification and to avoid convergence problems. In most cases, however, a variable transformation is then required to meet the model assumptions, at the expense of dealing with transformation bias. Nevertheless, transformation is a proven method in statistical modeling and is often used to linearize the relationships between dependent and independent variables, to homogenize the variance of residuals and to normalize regression residuals. A failure to meet the second and third requirements will not cause bias in the model estimates but will, however, reduce the reliability of significance tests and the estimation of confidence intervals of the regression coefficients.

Figure 6 shows to what extent different values for λ affect the relationship between a variable y and the transformed form y'. If the transformation parameter λ lies at or near zero, the logarithmic transformation would be suitable (Equation 3). The dependent variable would then be scaled on a much smaller value range than would be the case for a transformation where the values for λ are between 0.3 and 0.6, as shown in the results presented above (Table 2). The confidence intervals of the transformation parameters show, however, that these are significantly different from zero in every case and that the logarithmic transformation therefore cannot be regarded as the better option (Table 2). The higher flexibility of the Box-Cox transformation leads to a clear improvement of the residual structure in the transformed mode and especially to a reduction of the skew (Figure 2).

The multiple fits to randomly chosen partial data sets enabled a determination of whether the two methods differed systematically or randomly. It could also be shown using statistical tests as well as through the graphical analysis of the cross-validation that the MSE was significantly lower when the Box-Cox transformation was used and therefore that the predictive accuracy was higher (Figure 4).

It was shown that the two transformation methods differed not only in evaluation statistics but also in application. The log-transformation models yield systematically lower estimates for the observed tree species (Figure 5), which leads to ever greater differences in the diameter values over time. In practice, differences of even a few centimeters could have an important impact because the dbh is the deciding parameter in the calculation of volume and biomass, in stand basal area, in timber grading, or in decisions on whether to harvest a tree (target diameter).

Serinaldi et al. (2012), in a comparison of different models or transformations for estimating single tree volume, found results different from those in the current study. The often used method of logarithmization of all terms on both sides of the equation proved to be superior in the description of allometric relationships. Serinaldi et al. (2012) used the Box-Cox transformation to normalize both dependent and independent variables, whereas in this study the parameter for left-sided transformation was determined iteratively by normalizing the model residuals. Furthermore, in their study, Serinaldi et al. (2012) compared the transformation methods solely on the basis of a single model fit. An analysis of the variability of the goodness-of-fit statistics, by cross-validation for example, was not carried out. A further example of the use of the Box-Cox transformation in forestry is the study by Eastaugh and Hasenauer (2011), who used it in the modeling of historical timber usage levels from inventory data. The only previous use in the field of growth modeling at the level of the individual tree can be found in Garcia (1983). The Box-Cox transformation was implemented in a modified Bertalanffy-Richards model to describe the height development of forest stands. Neither a residual analysis nor a comparison with other transformation methods was carried out.

Currently the use of logarithmic variable transformation prevails in other fields of forest growth modeling too, such as forecasting the height growth increment (e.g., Hasenauer and Monserud 1997, Fahlvik and Nyström 2006, Uzoh and Oliver 2006, Nunifu 2009) or crown size (e.g., Hasenauer 1997, Petersson 1997, Hein and Spiecker 2008). On the basis of the results presented here, it is recommended that the Box-Cox transformation should be considered as an alternative in statistical modeling if the transformation of variables is required. In the use of generalized models (generalized linear model or GAM), it is also advisable to carry out an additional Box-Cox transformation and to use the transformation parameter determined in the specification of the link function.

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